Linear Depreciation

Lesson four

Terminal Objective

Students will solve depreciation word problems by writing linear equations.

Content Standard Reference:

Algebra 5: Students solve multi-step problems, including word problems, involving linear equations and linear inequalities in one variable and provide justification for each step.

Algebra 8: Students understand the concepts of parallel lines and perpendicular lines and how their slopes are related. Students are able to find the equation of a line perpendicular to a given line that passes through a given point.

Materials
Linear Depreciation Powerpoint

Time Required
1 class

Introduction of Lesson

Anticipatory Set:
We can use linear equations to determine an object's value after years of use.

The Port of Long Beach invests millions of dollars in the building of infrastructure and purchasing of equipment. As it ages, infrastructure and equipment lose value or depreciate.

Student Objective:
Students will use linear equations to find the depreciation of equipment at the Port of Long Beach.

Purpose:
Mathematics is a symbolic language that we use to represent and study the world around us. In this lesson you will use algebra to model simple depreciation as used in business. Using math to study real world problems will provide a better understanding of the uses of math outside of the classroom.

Lesson

Keyword
1. **Slope** - is the rate of change of y with respect to x

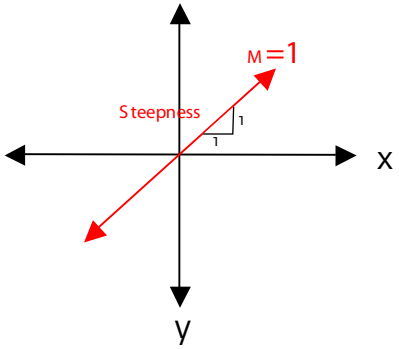
Input
Before students can model using a linear equation, they must remember how to write a linear equation. Begin by reviewing slope and two ways to use points to write a linear equation.

The ***slope*** is the rate of change of y with respect to x. Students can visualize slope as the steepness of the line. (Have them think of a really steep hill they would have to walk up or down)

Lesson cont'd

Modeling

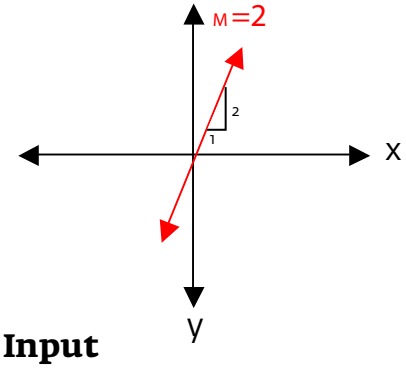
On this graph, with each single unit increase in x, there is a single unit increase in y. Slope = $\frac{\text{rise}}{\text{run}}$.



Input

As the absolute value of the slope becomes larger, the line becomes steeper, moving toward vertical.

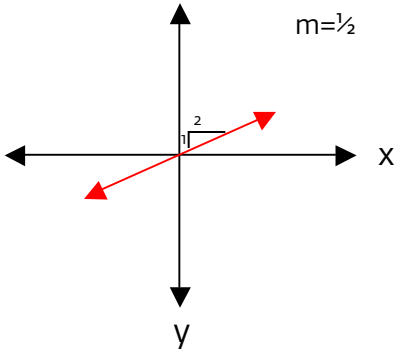
Modeling



Input

As absolute value of the slope becomes smaller, the line becomes flatter, moving toward horizontal.

Modeling



Lesson cont'd

Check for Understanding

Have students ask their neighbor:
The slope represents the _____ of a line.
Answer: steepness

As the absolute value of the slope becomes larger, _____. **Answer**: the line becomes steeper, moving toward vertical.

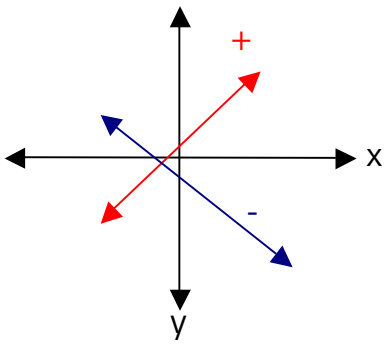
True or False: A line with slope of 1/8 will be flatter, moving toward horizontal. **Answer:** True

Input

A line with a positive slope is drawn up and to the right.

A line with a negative slope is drawn down and to the right.

Modeling



Input

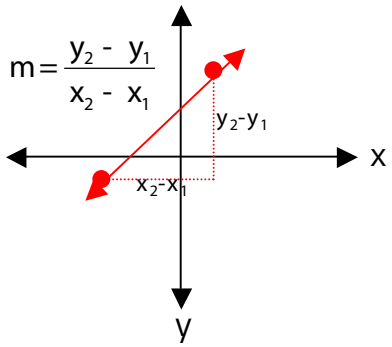
If points (x_1, y_1) and (x_2, y_2) are two points on a non-vertical line, then the slope of the line is given by the equation:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Lesson cont'd

Modeling

The slope is the change in the y over the change in the x.



Check for Understanding

Find the slope of the line that passes through the points (5, -7) and (2, 4).

$$m = \frac{4 - (-7)}{2 - 5}$$

$$m = \frac{11}{-3}$$

1. The slope of the line that passes through the given points is

   $$m = \frac{11}{-3}$$

2. This line is drawn _____. **Answer:** down and to the right

3. Is this line steep –going toward vertical, or is it flatter – going toward horizontal? **Answer**: steep –going toward vertical.

Guided Practice

Find the slope of the line that passes through the points (3,10) and (-3, 8).

Lesson cont'd

$$m = \frac{8 - 10}{-3 - 3}$$

$$m = \frac{-2}{-6}$$

$$m = \frac{1}{3}$$

Discuss the following questions with your neighbor:

Is this line drawn up or down to the right? **Answer**: Drawn up and right

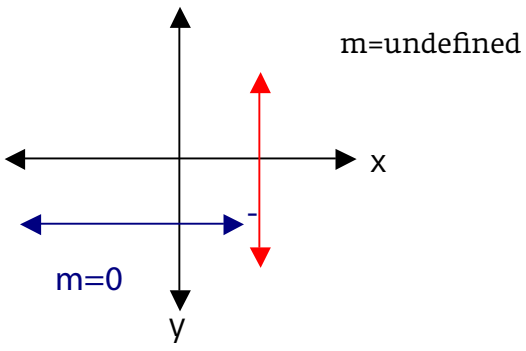
Is this line "steeper, going toward vertical" or is it "flatter, going toward horizontal"? **Answer**: flatter, going toward horizontal.

Input

The slope of a vertical line is undefined. If you select two points on a vertical line and solve for the slope, the result will be a zero in the denominator.

The slope of a horizontal line is zero. If you select two points on a horizontal line and solve for the slope, the result will be a zero in the numerator.

Modeling



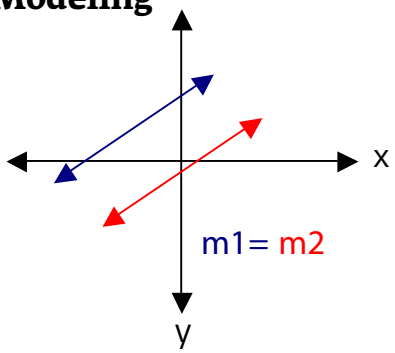
Lesson cont'd

Input

Two lines are ***parallel*** if they have the same slope, $m_1 = m_2$.

Two lines are ***perpendicular*** if the slopes of the two lines are negative reciprocals and their product is -1.

Modeling



Check for Understanding

L_1 passes through the points (1,-2) and (4,2), L_2 passes through the points (-1,-2) and (3,6). Determine whether lines are parallel, perpendicular or neither.

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1} \qquad m_2 = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m_1 = \frac{2 - (-2)}{4 - 1} \qquad m_2 = \frac{6 - (-2)}{3 - (-1)}$$

$$m = \frac{4}{3} \qquad m_2 = \frac{8}{4}$$

$$m_2 = 2$$

$$m_1 \neq m_2$$

The slope of L_1 is 4/3 and the slope of L_2 is 2. Are these lines parallel?

Raise your right hand if you think the answer is yes, and your left if you think the answer is no.

Answer: No, the slopes are not the same.

Keywords

1. **Parallel** - two lines with the same slope
2. **Perpendicular** - slopes of two lines are negative reciprocals and their product is -1

Lesson cont'd

Guided Practice

L_1 passes through the points (-2,5) and (4,2), L_2 passes through the points (-1,-2)and (3,6). Determine whether lines are parallel, perpendicular or neither.

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1} \qquad m_2 = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m_1 = \frac{2-5}{4-(-2)} \qquad m_2 = \frac{6-(-2)}{3-(-1)}$$

$$m_1 = \frac{-3}{6} \qquad m_2 = \frac{8}{4}$$

$$m_1 = -\frac{1}{2} \qquad m_2 = 2$$

The slope of L_1 is -1/2 and the slope of L_2 is 2.
Are these lines parallel? **Answer**: No.
Why? **Answer**: The slopes are not the same.

Check for Understanding

The slope of L_1 is 4/3 and the slope of L_2 is 2.

Are these lines perpendicular?

Raise your right hand if you think the answer is yes, and your left if you think the answer is no.

Answer: No, they are not negative reciprocals.

Lines L_1 and L_2 are neither parallel nor perpendicular.

To be parallel, the slopes have to be equal.

To be perpendicular, the slopes have to be negative reciprocals (the product of the two slopes will equal -1).

Lesson cont'd

Explain to your neighbor how you determine whether two lines are perpendicular or parallel.

Guided Practice

The slope of L_1 is -1/2 and the slope of L_2 is 2.
Are these lines perpendicular?

$$-\frac{1}{2} \times 2 = -1$$

Answer: Yes

Why? **Answer**: The product of the two slopes is -1. The numbers are negative reciprocals.

Input

We can write the equation of a line if we know two points on the line or a point on the line and the slope of the line.

We can use the Point–Slope Form which is given by the equation:

$y - y_1 = m(x - x_1)$

Modeling

Find the equation of the line that passes through the points (4, 5) and (6, -1).

Step 1: Find the slope.

$$m = \frac{-1-5}{6-4}$$

$$m = \frac{-6}{2}$$

Step 2: Using point (4,5) and m= -3, substitute the values into the equation.

$y - y_1 = m(x - x_1)$.

Lesson cont'd

$y - 5 = -3(x - 4)$
$y - 5 = -3x + 12$
$y = -3x + 17$

Check for Understanding

Find the equation of the line that passes through the point (2,3) and (-4, -6) using the Point Slope equation, $y - y_1 = m(x - x_1)$

$$m = \frac{-6 - 3}{-4 - 2}$$
$$m = \frac{-9}{-6}$$
$$m = \frac{3}{2}$$

$y - y_1 = m(x - x_1)$
$y - 3 = \frac{3}{2}(x - 2)$
$y - 3 = \frac{3}{2}x - 3$
$y = \frac{3}{2}x$

Guided Practice

Write the equation of the line passing through the points (5,1) and (-6, -4).

$$m = \frac{-4 - 1}{-6 - 5}$$
$$m = \frac{-5}{-11}$$
$$m = \frac{5}{11}$$

$y - y_1 = m(x - x_1)$
$y - 1 = \frac{5}{11}(x - 5)$
$y - 1 = \frac{5}{11}x - \frac{25}{11}$
$y = \frac{5}{11}x - \frac{25}{11} + 1$
$y = \frac{5}{11}x - \frac{25}{11} + \frac{11}{11}$
$y = \frac{5}{11}x - \frac{14}{11}$

Lesson cont'd

Input
Sometimes we are given the slope and a point.

Modeling
Find the equation of the line that passes through the point (6, -2) and has a slope of 2.

$y - (-2) = 2(x - 6)$
$y + 2 = 2x - 12$
$y = 2x - 14$

Check for Understanding
Write the equation of the line with a slope of zero and passing through the point (6,9).

$y - y_1 = m(x - x_1)$
$y - 9 = 0(x - 6)$
$y - 9 = 0$
$y = 9$

Is this line vertical or horizontal? Why? **Answer:**
It is horizontal because the slope is zero.

Input
Another way to find the equation of a line is by using the slope intercept form:

$$y = mx + b$$

Given two points we can find the slope of the line.
With the slope and a point we solve for b.
Once we have m and b we substitute the values in to the equation.

Modeling
1. Find the slope of points (1,3) and (4,-6).

Lesson cont'd

$$m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$
$$m = \frac{-6 - 3}{4 - 1}$$
$$m = \frac{-9}{3}$$
$$m = -3$$

2. Find b

$$y = mx + b$$
$$3 = -3(1) + b$$
$$3 = -3 + b$$
$$6 = b$$

3. Substitute

$$y = mx + b$$
$$y = -3x + 6$$

Check for Understanding

Using the slope intercept form of the linear equation, find the equation of the line that passes through the point (4,1) and (5,-3).

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$m = \frac{-3 - 1}{5 - 4}$$
$$m = -4$$

$$y = mx + b$$
$$1 = -4(4) + b$$
$$1 = -16 + b$$
$$b = 17$$

$$y = mx + b$$
$$y = -4x + 17$$

Guided Practice

Find the equation of the line passing through the given points. Use the specified method.

Lesson cont'd

A) Point-Slope, (4,7) and (-2, -3).

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{-3-7}{-2-4}$$

$$m = \frac{-10}{-6}$$

$$m = \frac{5}{3}$$

$$y - y_1 = m(x - x_1)$$

$$y - 7 = \frac{5}{3}(x - 4)$$

$$y - 7 = \frac{5}{3}x - \frac{20}{3}$$

$$y = \frac{5}{3}x - \frac{20}{3} + 7$$

$$y = \frac{5}{3}x - \frac{20}{3} + 7(\frac{3}{3})$$

$$y = \frac{5}{3}x - \frac{20}{3} + \frac{21}{3}$$

$$y = \frac{5}{3}x + \frac{1}{3}$$

B) Slope – Intercept for (-4, -6) and (5, 8).

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{8-(-6)}{5-(-4)}$$

$$m = \frac{14}{9}$$

$$y = mx + b$$

$$-6 = \frac{14}{9}(-4) + b$$

$$-6 = \frac{-56}{9} + b$$

$$-6 + \frac{126}{9} = b$$

$$-6(\frac{9}{9}) + \frac{56}{9} = b$$

$$\frac{-54}{9} + \frac{56}{9} = b$$

$$b = \frac{2}{9}$$

$$y = \frac{14}{9}x + \frac{2}{9}$$

Input

Students will be writing linear equations for the straight line method of linear depreciation to study the depreciation of capital investments made by the Port of Long Beach.

Depreciation is the decrease or loss in value of capital due to age, wear or market conditions. In accounting it is the allowance made for a loss in the value of capital.

Keyword

1. Depreciation - the decrease or loss in value of capital due to age, wear or market conditions

Lesson cont'd

Modeling

In 2002 the Port of Long Beach purchased a ZPMC Crane for Pier T for the amount of $6,811,461.73. The crane is to be depreciated over 15 years with a scrap value of $0. Write an expression that will calculate the value of the crane at the end of year (t). What is the value of the crane in 2007?

Two coordinates of the form (time, value).
(0, $6,811,461.73)
(15, $0)

$$m = \frac{(0 - 6,811,461.73)}{(15,0)}$$

$$m = \frac{-6,811,461.73}{15}$$

$$m = -454,097.45$$

To find the equation, use the slope and one of the points.

$y - y_1 = m(x - x_1)$
$y - 0 = -454,097.45\ (x - 15)$
$y = -454,097.45x + 6,811,461.73$

To find the price in 2007, subtract 2007-2002 to find the number of years of depreciation.

$2007 - 2002 = 5$

$y = -454,097.45(5) + 6,811,461.73$
$y = \$4,540,974.48$

Lesson cont'd

Check for Understanding

In 1984, a tractor/loader was purchased for use at the Port of Long Beach for a price of $29,041.01. The tractor/loader was depreciated using the straight-line method over 8 years. Find the linear equation expressing the tractor's book value at the end of x years. What is the rate of depreciation?
(0, 29,041.01) and (8,0)

$$m = \frac{0 - 29{,}041.01}{8 - 0}$$
$$m = -3630.13$$

$$y - y_1 = m(x - x_1)$$
$$y - 0 = -3630.13(x - 8)$$
$$y = -3630.13x + 29{,}041.01$$

Check for Understanding

The linear equation expressing the crane's value at the end of x years is given by

y= -3,630.13x + 29,041.01

What is the rate of depreciation?
-3,630.13.
What was the value of the crane in 1987? (1987-1984=3)
y=-3,630.13x + 29,041.01
y=-3,630.13(3)+29,041.01
y=18,150.62

Guided Practice

A truck scale purchase at a cost of $151,999.75 in 1986 has a scrap value of $0 at the end of 10 years.
(0,151,999.75) and (10,0)

If the straight-line method of depreciation is used,

Lesson cont'd

A) Find the rate of depreciation.

m= -15,199.98

B) Find the linear equation expressing the book value of the scale at the end of x years.

y= -15,199.98x + 151,999.75

C) Find the book value at the end of 7 years.

y = 45,600.03

Closure

Write a brief paragraph explaining the method for writing simple depreciation equations. Include an explanation of depreciation.

Share your paragraph with your neighbor.

Worksheet

Lesson four

Linear Depreciation Student Worksheet

Solve

Find the slope of the line that passes through the given points and determine if the lines are parallel, perpendicular or neither.

1. L_1 (1, - 2) and (-3, 10)
   L_2 (1,5 and (-1, 1)

2. L_1 (-2, 5) and (4, 2)
   L_2 (-1, -2) and (3,6)

3. Find the equation of a horizontal line that passes through the point (-5, 2).

4. Find an equation of the vertical line that passes through the point (-4, 5).

5. Find an equation of the line in slope-intercept form that passes through the point (3, -4) and has a slope of 5.

6. Find an equation of a line that passes through the points (-1, 3) and (2, 9). Put the equation in slope-intercept form.

7. Find an equation of a line in slope-intercept form that passes through the points (6, -2) and (4, 7) using the Slope-Intercept equation.

8. In 1983, the Port of Long Beach built the Arco Oil Terminal for a cost of $8,040,683.08 to be depreciated over its useful life of 40 years. Given the scrap value is $0, write the equation representing the linear depreciation of the terminal. What is the rate of depreciation? What is the book value of the terminal at the end of 2007?

9. The NH Terminal Railroad was built in 2002 for a cost of $1,084,139.68. The railroad will be depreciated over 15 years to a final book value of $0. Write the linear expression that represents the value of the railroad at time (t). What is the rate of depreciation? What will be the book value of the railway in 2010?

10. What is depreciation? Describe how you applied algebra to the above world problems to derive the linear depreciation equation.

Answers

Lesson four

Linear Depreciation Student Worksheet

1. $L_1 = \frac{10 - (-2)}{-3 - 1}$

$L_1 = \frac{12}{-4}$

$L_1 = -3$

$L_2 = \frac{1 - 5}{-1 - 1}$

$L_2 = \frac{-4}{-2}$

$L_2 = 2$

L1 and L2 are neither parallel nor perpendicular.

2. $L_1 = \frac{2 - 5}{4 - (-2)}$

$L_1 = \frac{-3}{6}$

$L_1 = -\frac{1}{2}$

$L_2 = \frac{6 - (-2)}{3 - (-1)}$

$L_2 = \frac{8}{4}$

$L_2 = 2$

The lines are perpendicular as the slopes of the lines are negative reciprocals.

3. y = 2.

4. x = -4

5. $m = 5$

(3,-4)

$y - y_1 = m(x - x_1)$

$y - (-4) = 5(x - 3)$

$y + 4 = 5x - 15$

$y = 5x - 19$

6. $m = \frac{9 - 3}{2 - (-1)}$

$m = \frac{6}{3}$

$m = 2$

$y - y_1 = m(x - x_1)$

$y - 3 = 2(x - (-1))$

$y - 3 = 2x + 2$

$4 = 2x + 5$

7. $m = \frac{7-(-2)}{4-6}$

$m = \frac{9}{-2}$

$y = mx + b$

$y + 2 = -\frac{9}{2}(x-6)$

$2y + 4 = -9(x-6)$

$2y = -9x + 54 - 4$

$2y = -9x + 50$

$y = -\frac{9}{2}x + 25$

9. Coordinates
(0, 1,084,139.68) and (15, 0)

$m = \frac{0-1{,}084{,}139.68}{15-0}$

$m = -72{,}275.98$

$y - y_1 = m(x - x_1)$

$y - 1{,}084{,}139.68 = -72{,}275.98(x-0)$

$y - 1{,}084{,}139.68 = -72{,}275.98x$

$y = -72{,}275.98x + 1{,}084{,}139.68$

The rate of depreciation is $72,275.98 per year.

The book value of the railway in 2007 will be $505,931.80.
Y=-72,275.98 (8)+1,084,139.68
Y=505,931.80

8. Coordinates:
(0, 8,040,683.08) and (40, 0)

$m = \frac{0 - 8{,}040{,}683.08}{40 - 0}$

m = - 201,017.08

$y - y_1 = m(x - x_1)$

y - 8,040,683.08 = - 201,017.08(x - 0)

y - 8,040,683.08 = - 201,017.08x

y = - 201,017.08x + 8,040,683.08

The rate of depreciation is $201,017.08 per year.

The book value of the terminal in 2007 is y = -201,017.08(24)+8,040,683.08
y=$3,216,273.16

10. When a company depreciates an asset, it is accounting for the loss of value of the item for business purposes.

Algebra was applied in the calculation of the linear equation by first determining the coordinates to use for the calculation of the slope. Once we had calculated the slope of the line, we then used the point-slope formula to derive the equation.